Diskussionspapiere Discussion Papers

Discussion Paper No. 202

Demographic Change, Endogenous Labor Supply and the Political Feasibility of Pension Reform

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Berlin, February 2000

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Internet: http://www.diw.de
ISSN 1433-0210

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Abstract

Options for reforming unfunded public pension schemes that are now being discussed all share the feature that the burden induced by demographic change would be shifted towards presently living and away from unborn generations. Existing models of the political economy of pension reform can not explain why such reform options are being discussed at all. We present an alternative model in which the possibility of evasion of workers from payment of social security taxes is taken into account by modelling a labor supply function. It turns out that the burden of demographic change may fall completely or at least predominantly on the pensioners. Thus this type of model can much better explain recent trends in legislature on unfunded public pension systems in industrial democracies.

Keywords: Public Pensions, Endogenous Labor Supply, Demographic Change

JEL-Classification: H55, J22, J18

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♣ This work was completed while the first author was visiting the Research School of Social Sciences, Australian National University. The authors gratefully acknowledge valuable comments by Simon Grant, Franco Perrachi, participants of seminars at Tilburg University, the Australian National Uiversity, the University of New South Wales and the University of Adelaide and to an anonymous referee.

1 Introduction

Reforms of the existing public pension systems are now being discussed in many industrial nations. Especially in countries with pronounced below-replacement fertility like Italy and Germany, the present mode of pay-as-you-go financing is seen by many observers as not suited to provide an adequate level of retirement income at acceptable costs for the working generation as soon as the "baby-boom" generation starts retiring in the third decade of the next century. Some are even convinced that a radical change of the system is urgent because they see it as already "on the verge of collapse" (cf. Börsch-Supan 1998).

Even in the political arena, rapidly rising social security tax rates have prompted politicians in several countries like Sweden, Germany and the United Kingdom to propose and enact major changes in the benefit formula although radical reforms of the financing system such as the abolition of earnings-related pensions in favor of a uniform (and low) pension for all or even to a capital-reserve system is supported by only a handful, such as the prime minister of the German Land of Saxony, Kurt Biedenkopf.

In addition to these casual observations, the empirical evidence from 20 OECD countries shows that when due to declining past birth rates the ratio of the aged (over 60) to the middle aged (40-60) rises, the overall size of the program increases as well (Breyer/Craig 1997), which means that under demographic ageing any cuts in the benefit formula are not strong enough to stabilize the tax rate completely.

What is not so clear, however, is whether substantial reforms could meet with the approval of the majority of voters. Theoretical models of voting on the level of contributions and benefits in an unfunded pension system, starting with Browning (1975), suggest the political acceptance of the system even when its implicit rate of return falls short of the corresponding figure in a capital-reserve system. This result, which is pretty robust with respect to the formulation of the model, is due to the high proportion of voters in or near retirement age who will benefit from the maintenance of the system even if they would have opposed it when they were young[1]. Moreover, as the proportion of older people in the electorate is higher the lower the fertility rate and thus the implicit return of the pay-as-you-go system, the models yield the seemingly paradox result that a negative rate of return makes the system politically more stable instead of less, which Marquardt and Peters (1997) interpret as "collective madness".

Thus there is an apparent conflict between theoretical predictions derived from models of rational voter behavior and the observable facts of widespread attempts by politicians to at least cut the size of the pension systems when the worker-pensioner ratio is shrinking. The

[1] For as survey of the vast literature on the political economy of public pensions until 1993 see Breyer (1994a).

present paper tries to resolve this conflict by proposing an alternative theory of political decision-making which is able to explain the observed facts better than existing models. It based on two key elements: first, the inability of present voters to commit future voters and taxpayers, and second, the adverse incentive effects of social security taxes on the labor supply of the working generation[2]. It will be shown that under these assumptions, the costs of a decline in fertility will be borne partly or even fully by the pensioners through cuts in the level of retirement benefits.

The paper is organized as follows. Section 2 summarizes the results of previous theoretical models of voting on a pension reform in response to demographic ageing and in Section 3 our own model is proposed. Section 5, finally, contains some concluding remarks.

2 Existing Models of the Political Economy of Ageing and Pension Reform

Among the large number of models of political decision-making on the size of an unfunded pension system only a few contributions explicitly address the comparative static effects of a change in the worker-pensioner ratio due to a decline in fertility.

2.1 A Model of once-and-for-all decisions on benefit level

In the simplest case it is assumed that voters believe that their decision on the level of the contribution rate will be binding forever. Individual rationality then implies that, in choosing among several alternatives, each person votes for that one which promises him/her the highest utility for his/her remaining lifetime (see, e.g. Browning 1975). The introduction or expansion of a pay-as-you-go financed pension system benefits not only the present pensioners but also those cohorts that are close to retirement. As they view the outcome of the voting as permanent, they stand to gain from a high tax rate for a longer period than the one in which they have to bear an additional burden, and thus the discounted value of retirement benefit claims exceeds that of their remaining contributions. Consequently, the older a voter at the time of voting, the higher the optimal tax rate, and preferences of all voters are single-peaked. The "political equilibrium" is then the optimal tax rate for the voter of median age, in general a worker in the second half of his working life, and this tax rate will be positive even if the rate of return offered by the pay-as-you-go system, i.e. the growth rate of labor incomes, falls short of the interest rate. In this type of model, if population grows more slowly (or shrinks faster), the age of the median voter increases. This effect by itself would lead the equilibrium contribution rate to rise[3]. On the other hand, shrinking population growth reduces the rate of return of the pay-as-you-go method. This effect alone makes everybody prefer a

[2] The latter aspect was already stressed in Breyer (1994b).
[3] This effect is empirically confirmed in a regression analysis fro OECD countries, cf. Breyer/Craig (1997).

smaller contribution rate if the substitution effect dominates the income effect of the change. Therefore, the total effect is indeterminate.

In a recent contribution, Marquardt/Peters (1997) have presented a continuous-time model in which the lifetime is not certain, but individuals face an exogenous and constant risk of death, while the birth rate and the interest rate in the perfect capital market are also exogenous. Retirement age is given, labor supply is exogenous, but social security taxes and transfers are subject to a deadweight loss, which is a convex function of the size of the program[4]. In comparing different steady states, the authors give sufficient conditions under which a fall in the birth rate leads to an increase in the equilibrium contribution rate. Their characterization of such an outcome as "collective madness" is not quite convincing, though, because in order to restore the social security budget equation after a drop in the birth rate, it is straightforward that either the contribution rate must rise or the benefit level must fall or both.

What is more remarkable, however, is that the Marquardt/Peters model can generate equilibria in which the effect of the rise in median voter age is so dominant, that not only the tax rate but also the benefit level rises when the birth rate falls. If the term "collective madness" is at all justified, then perhaps in such a case.

2.2 A Model of repeated decision-making

The conception of a permanently valid decision is unsatisfactory as in a democracy the electorate always has the power to enact even radical reforms of an existing pension system. Therefore it is of great interest to be able to explain why so many pay-as-you-go financed pension schemes have survived over several generations, and it is also important to forecast whether and how they will persist given the huge demographic imbalance due in the 21. century. As the answer to this question appears to be obvious in the case in which the majority of the electorate consists of pensioners, theoretical models within this branch of the literature have assumed alternatively, a) that the majority rule is applied (direct democracy) and the majority of voters are working-age people, or b) that there is an indirect democracy in which politicians maximize a weighted sum of the utility functions of a representative worker and a representative pensioner, where the (relative) weight of a worker depends upon the share of workers in the electorate and thus upon the population growth rate (Verbon/Verhoeven 1992).

The sequence of decisions on the structure of the pension system for the following period (generation) can then be modelled as a sequential game with infinitely many players (politicians), each of whom maximizes a weighted sum of utilities of a worker and a pensioner at the respective time period by choosing the appropriate level of the contribution

[4] Otherwise, anyone who is in favor of a positive social security tax rate, would prefer a larger to a smaller one so that an equilibrium would not exist.

rate for the present period. The contribution rate chosen by earlier and by later generations of decision-makers are taken as given (Cournot-Nash-behavior).

Within this framework the authors analyze the time path of adjustment of the contribution rate before and after a "demographic shock", i.e. a change in the population growth rate, assuming perfect foresight. It turns out that a one-time drop of fertility can lead to a temporary increase or decrease of the rate of contributions - depending on whether the political weight of a worker (in relation to the one of a pensioner) exceeds 1 and how this weight reacts to a variation of the fertility rate. The most paradox result within this branch of the literature is that the expectation of a permanent decline in fertility can lead a society to the introduction of an unfunded pension system when there has been none in the past (Meijdam/Verbon 1996a,b).

An important weakness of this type of model in terms of their ability to forecast the future development of unfunded pension systems is that a crucial explanatory variable, the political weight of a worker, can not be observed in the real world and thus a prognosis can not be derived using available data. Moreover, like the Marquardt/Peters model described above this model can not explain the recent trend of cutting benefits when the demographic situation deteriorates.

3 A Model Based on Endogenous Labor Supply

3.1 General Assumptions

In contrast to the models discussed in Section 2, we do not view the political process in a democracy as an institution for finding compromises between the members of different generations. We rather consider it as a sequential game in which the majority group behaves like a decision maker who maximizes his utility under constraints which are given through the reactions of the minority to the rules imposed by the majority. So the model follows the tradition e.g. of Meltzer/Richard (1981), who explain the limits to redistributive taxation by the labor disincentives that an income tax provides for the high-wage classes (see also Breyer 1994b).

In our case of intergenerational transfers in a society with low fertility it can not be ignored that the majority of the electorate is either already retired or will do so in the near future[5]. Therefore, within the framework of a two-overlapping-generations model it is justified to assume that it is the pensioners who have the power to determine the contribution

[5] By the year 2050, the fraction of all people in voting age (20 and over) who are already over 55 will reach, e.g., 52.2 per cent in Italy, 52 per cent in Spain and 49.9 per cent in Germany (Eurostat 1996, p. 210).

rate, while the workers can only react to it by varying their labor supply accordingly. In particular, the model is built on the following assumptions:

A1. Every individual lives for two periods, the first as a worker, the second as a pensioner.

A2. Population size develops exogenously: the number N_t of workers in period $t \in Z$ (which by assumption A1 equals the size of generation t) is described by an infinite sequence of growth factors n_t, where $n_t = N_t / N_{t-1}$.

A3. The country is a small open economy, i.e. there is an exogenously given infinite sequence of interest rates $r_t > 0$, $t \in Z$. For sake of simplicity we assume constant interest rate, $r_t \equiv r$.

A4. There is no technological progress so that the wage rate is constant over time and can thus be normalized to 1.

A5. (a) The representative worker of generation t determines his labor supply $\ell_t \in [0,1]$ (as a fraction of total available time) and his savings $s_t \in R$ so as to maximize his utility function $U_t = U(c_t, z_{t+1}, 1-\ell_t)$ subject to the budget constraints

$$c_t = (1-\tau_t)\ell_t - s_t$$

$$z_{t+1} = (1+r)s_t + p^e_{t+1},$$

where c_t and z_{t+1} denote his working-age and retirement consumption, respectively, $\tau_t \in [0,1]$ denotes the contribution rate in period t and p_{t+1} the benefit level of public pensions that the representative worker of generation t expects for period $t+1$.

(b) Expected pension benefits are an exogenous parameter in the representative worker's utility maximization problem:

$$p^e_{t+1} = \overline{p}_{t+1}$$

(c) For any $(\tau_t, \overline{p}_{t+1})$ in $[0,1] \times R_{\geq 0}$ the utility-maximization described in (a) yields a unique labor supply $\ell_t = \ell(\tau_t, \overline{p}_{t+1})$ with $\ell(1,.) \equiv 0$.[6]

We refer to $\ell : [0,1] \times R_{\geq 0} \to [0,1]$, $(\tau, \overline{p}) \mapsto \ell(\tau, \overline{p})$ as the *labor supply function*.

A6. In each period t the pensioners know the representative worker's reaction function $\ell^r_t(\tau_t) := \ell(\tau_t, \overline{p}_{t+1})$ and choose the rate of contributions τ_t so as to maximize their pension claim

$$p_t = n_t \cdot \tau_t \cdot \ell^r_t(\tau_t).$$

In case this maximization problem has a solution, i.e. if the expected pension level $\overline{p}_{t+1}$ is an element of

[6] Because the interest rate is a constant, we omit r as a functional argument.

$$\mathrm{P} := \{\overline{p} \geq 0 \mid \max\{n_t \cdot \tau_t \cdot \ell(\tau_t, \overline{p}) \mid \tau_t \in [0,1]\} \text{ exists}\}, \tag{1}$$

we write $p°(\overline{p}_{t+1}, n_t)$ for the resulting pension level and refer to

$$p°: \mathrm{P} \times R_{>0} \to R_{\geq 0}$$
$$(\overline{p}, n) \mapsto p°(\overline{p}, n)$$

as the *pension function*.

Assumption A5 can be justified in the following way: Instead of supplying work to the official labor market in which the earnings are subject to the social security payroll tax, workers of generation t could switch to one or several alternatives, e.g.

- home production and increased leisure,
- working in the (untaxed) underground economy,
- commuting to work across the national border, where the payroll tax rate may be lower,

all of which become relatively more attractive when the social security tax rate τ_t is raised[7]. This modelling presupposes that the full amount of the contribution is viewed by the workers as a tax, which means that they do not perceive any connection to retirement benefits they can claim in their own old age. This perception certainly runs counter to the official rules of obtaining benefit claims valid in many pension systems such as the German one. However, it is consistent within this model because the extent to which any benefit claims would be honored by the next generation $t+1$ depends crucially on the future ability of the members of generation t to impose taxes on the latter. Thus a contribution-benefit linkage, even if it existed, could be only relative anyway.

3.2 Existence of a Steady-state Equilibrium under Perfect Foresight

The main subject of this subsection is to prove that - provided population growth factors are constant over time and some technical prerequisities are met - there is a steady-state equilibrium to the model presented above. To illustrate our general assumptions and results we develop in parallel a prominent example:

Example

In case the workers' utility function is of Cobb-Douglas-type

$$U(c, z, 1-\ell) = c^{\alpha} \cdot z^{\beta} \cdot (1-\ell) \quad \text{with } \alpha, \beta > 0$$

the first-order conditions of the utility maximization problem yield

[7] Although we did not explicitly model the possibility to work in the underground economy or abroad, the utility maximization problem could be easily extended in this direction.

$$\ell = \ell^{+}(\tau, \bar{p}) := \frac{\alpha+\beta}{1+\alpha+\beta}\left(1 - \frac{\bar{p}}{(\alpha+\beta)(1+r)(1-\tau)}\right)$$

whenever $\tau < 1$ and the right hand side is positive, else $\ell = 0$.[8] Hence assumption A5(c) is fulfilled and the labor supply function is

$$\ell(\tau, \bar{p}) = \begin{cases} \ell^{+}(\tau, \bar{p}) & \textit{if } \tau < 1 \textit{ and } \ell^{+}(\tau, \bar{p}) > 0 \\ 0 & \textit{otherwise.} \end{cases} \tag{2}$$

Now it's easy to see that $\ell(., \bar{p}) \not\equiv 0$ if and only *if* $\bar{p} < (\alpha + \beta)(1 + r)$ and that $\mathrm{P} = R_{>0}$.[9] The resulting pension function is

$$p^{\circ}(\bar{p}, n) = \begin{cases} n \cdot \dfrac{\alpha + \beta}{1 + \alpha + \beta}\left(1 - \sqrt{\dfrac{\bar{p}}{(\alpha + \beta)(1 + r)}}\right)^{2} & \textit{if } \bar{p} < (\alpha + \beta)(1 + r), \\ 0 & \textit{otherwise.} \end{cases}$$

Provided that expected benefit levels $\bar{p}_{t+1}$ are always positive (which means the pension system is never expected to break down completely) the model determines a unique sequence of benefit levels $p_t^{\circ} = p^{\circ}(\bar{p}_{t+1}, n_t)$.

To arrive at a meaningful equilibrium concept, we first have to endogenize the workers' expectations concerning future benefit levels.

3.2.1 Expectations

According to our model, the benefit level p_{t+1} in period t+1 will be determined by n_{t+1} and p_{t+2}^{e}, the benefit level of period t+2 expected by members of generation t+1. The benefit level p_{t+2}, in turn, is determined by n_{t+2} and p_{t+3}^{e} etc. and hence it seems plausible to assume that members of generation t will calculate their expected pension level based on the population growth factors they expect to appear in the future:

$$p_{t+1}^{e} = \bar{p}\left(n_{t+1}^{e,t}, n_{t+2}^{e,t}, n_{t+3}^{e,t}, \ldots\right)$$

where $n_s^{e,t}$ denotes the population growth rate generation t expects for period s.

3.2.2 Steady-state Equilibrium under Perfect Foresight

Now we restrict our attention to situations where population growth factors are constant over time and individuals have perfect foresight. By our assumptions we then have

[8] To be precise we have to assume $\ell = 0$ if $\tau = 1$ and $\bar{p} = 0$, because in this case the utility-maximising labor supply is not uniquely determined.
[9] In the case $\bar{p} = 0$ the maximization problem of the pensioners indeed fails to have a solution!

constant expected population growth rates $n_s^{e,t} = n_s \equiv n$ for all s and $t < s$, constant expected benefit levels $\overline{p}(n,n,n,...)$ and constant actual benefit levels $p°(n,\overline{p}(n,n,n,...))$. Perfect foresight furthermore requires that the expected meet the actual benefit levels, i.e $\overline{p}(n,n,n,...) = p°(n,\overline{p}(n,n,n,...))$ and the question arises, whether the equation $\overline{p} = p°(n,\overline{p})$ has a solution $p*$. But even if this is true we may not conclude $\overline{p}(n,n,n,...) = p*$ unless the solution is unique. This leads to:

Definition 1: *A steady-state equilibrium under perfect foresight consists of a benefit level $p*(n)$ and a contribution rate $\tau*$ such that $p*(n) \in \mathrm{P}$ is the unique solution of*

$$\overline{p} = p°(\overline{p}, n) \tag{3}$$

i.e. $p*(n) = n \cdot \tau * \cdot \ell(\tau*, p*)$, where n denotes the steady-state population growth factor.

For any n for which such a steady state exists, we assume that $\overline{p}(n,n,n,...) = p*$. This means, if constant future population growth rates n are perfectly foreseen by all generations $t \geq T$, then the steady state $p*(n)$ is actually realized in all periods $t \geq T$.

3.2.3 Existence

According to this definition a steady state exists if and only if equation (3) has a unique solution $\overline{p} = p*$. To guarantee this we assume:

A7. (a) The partial labor supply function $\ell(.,0)$ is not identically zero[10] and $\ell(.,\overline{p})$ is continuous whenever $\overline{p}$ is positive .

(b) For any $\tau \in [0,1]$ the partial labor supply function $\ell(\tau,.)$ is continuous and monotonically decreasing (i.e. leisure is a normal good).

Theorem 1: *Under assumptions A1-A7, for any value of n there exists a steady-state equilibrium under perfect foresight. The benefit level is always positive.*

Proof: see Appendix.

Example **(Cobb-Douglas, continued):**

The labor supply function calculated above fulfills assumption A7: Because

[10] Because $\overline{p}_{t+1} = 0$ implies that labor is expected to be the only source of income, this seems to be a natural assumption.

$$\ell(\tau,0)=\begin{cases}\dfrac{\alpha+\beta}{1+\alpha+\beta} & if\ \tau<1\\ 0 & if\ \tau=1\end{cases},$$

$\ell(.,0)$ is not identically zero, the desired continuity and monotonicity properties are also easily verified. So Theorem 1 implies the existence of a unique steady-state benefit level $p*$. A simple calculation yields

$$p*(n)=\frac{\alpha+\beta}{\left(\sqrt{\dfrac{1}{(1+r)}}+\sqrt{\dfrac{1+\alpha+\beta}{n}}\right)^2}. \tag{4}$$

4 Comparative statics and dynamics

4.1 Comparative Statics

So far we proved the existence of a steady state. To get uniqueness w.r.t. the contribution rate and clear-cut comparative static results we assume

A8. The labor supply function is twice continuously differentiable on the set $\Lambda^+ := \{(\tau,\overline{p}) \mid \ell(\tau,\overline{p})>0\}$. In addition, for any $\overline{p}\in \mathrm{P}^+$ there is exactly one $\tau^\circ=\tau^\circ(\overline{p})$ such that $n\cdot\tau^\circ\cdot\ell(\tau^\circ,\overline{p})=p^\circ(\overline{p},n)$.

Theorem 2: Under assumptions A1-A8, if $n_t\equiv n$ there is a unique steady state $(p^*,\tau*)$. The pension level $p*$ is strictly and inelastically increasing in n. In detail we have

$$\varepsilon_{p^*,n}=\frac{1}{1-\varepsilon_{\ell,\overline{p}}(\tau^*,p*)}.$$

Proof: see Appendix.

The intuition behind this result is the following: A permanent decline in the population growth rate *n* has two effects on the benefit rate of the social security system, a direct and an indirect one. The direct effect, holding labor supply constant, implies that the benefit rate declines in the same proportion as the population growth rate. The indirect effect works through changes in labor supply: As *n* and expected future benefits both decline, future workers become poorer and thus - leisure being a normal good - increase their labor supply, which, taken by itself, increases total benefits. Thus, on the whole the benefit rate declines by a smaller percentage than the population growth rate.

To determine the sign of $\partial\tau*/\partial n$ we have to sharpen assumption A8:

A8'. The labor supply function is twice continuously differentiable and $\partial^2 \ell / \partial \overline{p} \partial \tau < 0$ on the set Λ^+.

Theorem 3: *Under assumptions A1-A7 and A8' the steady-state contribution rate $\tau *$ is decreasing in n.*

Proof: see Appendix.

Intuitively, when the population growth rate declines and thus workers become poorer (see Theorem 2), they become more vulnerable for exploitation because, by assumption A8', their labor supply becomes less elastic and thus pensioners increase the rate at which they tax worker's labor income.

If this reasoning is reversed, we see that it is conceivable that when the fertility rate shrinks, both the benefit level and the tax rate decline in steady-state equilibrium. The drop in expected pension benefits as such makes a worker poorer, and if this makes his labor supply more rather than less elastic (i.e. if assumption A8' does not hold), lowering the tax rate might make him work more by a sufficient degree to restore budget balance in spite of the smaller number of workers per pensioner and the lower tax rate.

Example **(Cobb-Douglas, continued):**

Because ℓ equals ℓ^+ on Λ^+ and ℓ^+ is twice continuously differentiable on Λ^+ with

$$\frac{\partial^2 \ell^+}{\partial \overline{p} \partial \tau}(\tau, \overline{p}) = -\frac{1}{(1+\alpha+\beta)(1+r)(1-\tau)^2} < 0,$$

assumption A8' holds. So Theorem 3 applies and we obtain a unique steady state (p^*, τ^*) where p^* is inelastically increasing and τ^* is decreasing in n.

On the other hand these properties can be verified with the explicit result (p^*, τ^*) where p^* is described in equation (4) and

$$\tau^* = 1 - \frac{1}{1+\sqrt{\frac{(1+\alpha+\beta)(1+r)}{n}}}.$$

4.2 Dynamics: The Role of Expectations

In this subsection we finally make use of our comparative static results in order to analyze the behavior of our model in response to permanent or non-permanent shocks in fertility. All scenarios under consideration will share the following features:

- Initially, i.e. until period -1, the system is in its steady state with constant population growth rate *n*. Generation -1 expects this growth rate to last forever and hence expects a pension level $p_0^e = p^*(n)$.

- In period 1 a demographic shock occurs $(n_1 = n' < n)$ which may be permanent $(n_t = n' \textit{ for all } t \geq 1)$ or non-permanent $(n_t = n \textit{ for all } t \geq 2)$.
- "Post-shock generations" $t \geq 1$ do have perfect foresight concerning future population growth rates $n_2, n_3, ...$

The last assumption especially implies that a (new) steady state will be reached in period 2 at the latest. What turns out to be crucial is whether or not the demographic shock was foreseen by generation 0.

4.2.1 Results for a permanent decline in fertility

First we consider a once and for all decrease in fertility: $n_t = n' < n$ for all $t \geq 1$. Because generations $t \geq 1$ have perfect foresight, they will correctly anticipate the new steady-state pension level $p*(n')$ for their retirement age and the pension level will attain its new steady-state level $p*(n')$ in period 2. Furthermore generation $t = 1$ will adjust their labor supply according to the new (smaller) pension level. Consequently $\tau_1 = \tau*(n')$ and $p_1 = p*(n')$. What remains to be discussed is the situation in period 0. If generation 0 doesn't expect the smaller pension level for their retirement age, their labor supply equals the labor supply of their parent generation and hence $\tau_0 = \tau*(n)$ and $p_0 = p*(n)$.

t n_t	-2 n	-1 n	0 n	1 n'	2 n'
p^e_{t+1}	$p*(n)$	$p*(n)$	$p*(n)$	$p*(n')$	$p*(n')$
τ_t	$\tau*(n)$	$\tau*(n)$	$\tau*(n)$	$\tau*(n')$	$\tau*(n')$
p_t	$p*(n)$	$p*(n)$	$p*(n)$	$p*(n')$	$p*(n')$

Table 1: An unexpected permanent decline in fertility.

On the other hand, if the decrease in fertility was anticipated by generation 0, they will increase their labor supply according to their reduced pension claim, which will result in an increased `pre-shock' contribution rate $\tau_0 = \tau^*(n)$ and pension level' $p_0 = n/n' \cdot p*(n')$ which is greater than $p*(n)$ by Theorem 2.

t	-2	-1	0	1	2
n_t	n	n	n	n'	n'
p^e_{t+1}	$p^*(n)$	$p^*(n)$	$p^*(n')$	$p^*(n')$	$p^*(n')$
τ_t	$\tau^*(n)$	$\tau^*(n)$	$\tau^*(n')$	$\tau^*(n')$	$\tau^*(n')$
p_t	$p^*(n)$	$p^*(n)$	$n/n' \cdot p^*(n')$	$p^*(n')$	$p^*(n')$

Table 2: An expected permanent decline in fertility.

4.2.2 Results for a temporary decrease in fertility

Similar arguments apply to the case of a one period shock in fertility $n_1 = n' < n$ and $n_t = n$ for all $t \neq 1$. First, because generation $t = 1$ knows that the drop in fertility is only temporary, they will not adjust their expectations concerning future benefit levels and hence generation 0 will bear the full burden of the demographic shock by proportionally reduced pension claims: $p_1 = n'/n \cdot p^*(n) < p^*(n')$ by Theorem 2. If these reduced pension claims were not foreseen by generation 0 during their working-age, this will be the only deviation from the initial (and final) steady state.

t	-2	-1	0	1	2
n_t	n	n	n	n'	n
p^e_{t+1}	$p^*(n)$	$p^*(n)$	$p^*(n)$	$p^*(n)$	$p^*(n)$
τ_t	$\tau^*(n)$	$\tau^*(n)$	$\tau^*(n)$	$\tau^*(n)$	$\tau^*(n)$
p_t	$p^*(n)$	$p^*(n)$	$p^*(n)$	$n/n' \cdot p^*(n)$	$p^*(n)$

Table 3: An unexpected one-period shock in fertility.

If the shock was foreseen by generation 0, they will adjust their labor supply according to their reduced pension claim $p_1 = n'/n \cdot p^*(n)$, which makes them even more vulnerable for exploitation during their working-life: from $p_1 < p^*(n')$ and Assumption A8', the expanded labor supply will lead to a contribution rate $\tau_0 > \tau^*(n') > \tau^*(n)$.

t	-2	-1	0	1	2
n_t	n	n	n	n'	n
p_{t+1}^e	$p^*(n)$	$p^*(n)$	$n'/n \cdot p^*(n)$	$p^*(n)$	$p^*(n)$
τ_t	$\tau^*(n)$	$\tau^*(n)$	$\tau_0 > \tau^*(n')$	$\tau^*(n)$	$\tau^*(n)$
p_t	$p^*(n)$	$p^*(n)$	$p_0 > p^*(n)$	$n/n' \cdot p^*(n)$	$p^*(n)$

Table 4: An expected one-period shock in fertility.

4.3 Discussion

The model presented here analyzes a demographic situation in which (e.g. due to a shrinking population) each generation of pensioners has both the determination and the power to exploit the willingness to pay taxes of the subsequent generation as fully as possible. This willingness, in turn, depends upon the possibilities to evade (domestic) taxes by working outside the country or substituting leisure for consumption. As a consequence of the revenue-maximizing behavior of the pensioner generation, the effects of a drop in the fertility rate and thus in the corresponding worker/pensioner ratio depend crucially on the beliefs of current workers and pensioners on the future path of fertility.

If current decision-makers believe that the decline in the fertility rate is permanent and indicates a transition to a new steady state with more rapidly shrinking population, then expectations with respect to future benefit levels have to adjust and the analysis of Section 4.2.1 applies. As a result, the level of benefits per pensioner falls, and secondly, under fairly mild conditions, the optimum tax rate rises so that the burden of the demographic change is shared between each two successive generations. If, in contrast, the fertility rate is commonly expected to rise back to its old steady-state level after one period, then expectations do not adjust and the analysis of Section 4.2.2 is relevant. Here, the optimal contribution rate from the point of view of present pensioners (= voters) does not change so that the burden of adjustment is exclusively borne by the present generation of pensioners. In either case, if the drop in fertility was anticipated, the last "pre-shock" parent generation will be able to exploit the increasing labor supply of their less fertile children and their pensions will be even higher than in the initial steady state.

If the results of the model are applied to the demographic situation which is currently valid in low-fertility countries throughout Europe, it is crucial to find out about the beliefs in the societies on the durability of low fertility rates. Up until the early sixties, fertility rates were at or slightly above replacement levels in most European countries, and during the seventies there has been a very pronounced drop in net replacement rates (to below .7 in Germany, Italy and Spain). Since then the rates have been more or less stable at the new level. Now that the smaller cohorts start entering the labor force, we observe e.g. in Germany and

Sweden that the pension systems are undergoing a series of reforms through which the benefit levels are gradually reduced, but not as much as would be required to hold the contribution rate constant at the present level. Thus these observations are consistent with our model if the drop in fertility rates is believed to be permanent.

5 Concluding Remarks

We can summarize the findings of this paper in the following statements:

1. Options for reforming unfunded public pension schemes that are now being discussed all share the feature that the burden induced by demographic change would shift towards presently living generations and away from unborn generations.
2. Existing models of the political economy of pension reform can not explain why such reform options are being discussed at all. As the median voter is either an older worker or already retired, they predict that with rising ratio of pensioners to workers unfunded pension schemes should be expanded rather than cut.
3. If the possibility of evasion of workers from payment of social security taxes is taken into account by modelling a labor supply function, then it turns out that at least in the extreme case that only pensioners decide on the level of a social security tax, at least part of the burden of demographic change must be borne by pensioners in such a way that the pension level in the new long-run equilibrium falls. Moreover, under plausible conditions the contribution rate rises so that the demographic burden is shared between workers and pensioners. Thus this type of model can much better explain recent trends in legislature on unfunded public pension systems in industrial democracies.

A limitation of the analysis in this paper has been the assumption of the absence of a linkage between taxes and benefits on the individual level. Thus the results strictly apply only to countries like the Netherlands with a flat-benefit pension scheme. Introducing a tax-benefit linkage would seem to be an interesting topic for future research. As was shown in Breyer (1994b), the equilibrium tax rate is always higher with a tax-benefit linkage than without, but this does not imply that it necessarily reacts to changes in fertility in the same way. We do not believe, however, that the results derived here would fundamentally change. After all, in dynamically efficient economies at least part of the social security contributions must be considered as a pure tax by the worker, and thus the same mechanisms that are at work in our model should apply there, too.

Appendix

Proof of Theorem 1

Lemma 1 *Under assumptions A1 - A7 we have:*

1. *If* $n_t \equiv n > 0$ *and* (p^*, τ^*) *is a steady state then*
$$p^* \in \mathrm{P}^+ := \{\overline{p} \in \mathrm{P} \mid \overline{p}, p°(\overline{p}, n) > 0\};$$
2. $R_{>0} \subset \mathrm{P}$ *and* $p°(.,n)$ *is continuous on* $R_{>0}$;
3. P^+ *is a nonempty open interval of the kind* $(0, \hat{p})$ *with* $\hat{p} \in R \cup \{\infty\}$;
4. $p°(.,n)$ *is monotonically decreasing on* P^+.

Proof:

1. By part (a) of assumption A6 we have $\ell(.,0) \not\equiv 0$, so $\max\{n \cdot \tau \cdot \ell(\tau,0) \mid \tau \in [0,1]\}$ either does not exist or $0 < \max\{n \cdot \tau \cdot \ell(\tau,0) \mid \tau \in [0,1]\} = p°(0,n)$. Consequently either $0 \notin \mathrm{P}$ or $0 \neq p°(0,n)$ and hence there is no steady state $(0, \tau^*)$.

2. Because by assumption A7(a) $\ell(.,\overline{p})$ is continuous if $\overline{p} > 0$, the first statement is clearly true. Moreover, because $\ell(\tau,.)$ is continuous for any $\tau \in [0,1]$ by assumption A7(b), the second statement is an immediate consequence of the well-known Theorem of the Maximum.

3. Because $\ell(.,0) \not\equiv 0$ by assumption A7(a) and $\ell(\tau,.)$ is continuous for any $\tau \in [0,1]$ by assumption A7(b), we have $\ell(.,\overline{p}) \not\equiv 0$ for sufficiently small $\overline{p} > 0$. Together with $R_{>0} \subset \mathrm{P}$ and assumption A7(b) this proves the claim.

4. Finally consider $\overline{p}_1, \overline{p}_2 \in \mathrm{P}^+$ with $\overline{p}_1 < \overline{p}_2$. Choose τ_2° such that $p°(\overline{p}_2, n) = n \cdot \tau_2^\circ \cdot \ell(\tau_2^\circ, \overline{p}_2)$. Normality of leisure implies $\ell(\tau_2^\circ, \overline{p}_1) \geq \ell(\tau_2^\circ, \overline{p}_2) > 0$ and hence $p°(\overline{p}_1, n) \geq n \cdot \tau_2^\circ \cdot \ell(\tau_2^\circ, \overline{p}_1) \geq n \cdot \tau_2^\circ \cdot \ell(\tau_2^\circ, \overline{p}_2) = p°(\overline{p}_2, n)$. ■

Now we are ready to prove the main theorem:

Theorem 1: *Under assumptions A1-A7, for any value of n there exists a steady-state equilibrium under perfect foresight. The benefit level is always positive.*

Proof: By (3) and Lemma 1 it suffices to show that there exists a benefit level $p^* > 0$ such that $p^* - p°(p^*, n)$. This benefit level will be unique, because $p°(.,n)$ is decreasing on P^+ by part 4 of lemma 1.

Consider Δ: $R_{>0} \to R, \overline{p} \mapsto \overline{p} - p°(\overline{p}, n)$. By part 2 of Lemma 1 Δ is well defined and continuous and parts 3 and 4 of this lemma guarantee the existence of a $\overline{p} > 0$ such that $\Delta(\overline{p}) < 0$. On the other hand, because $p°(\overline{p}, n) \leq n \cdot 1 \cdot 1 = n$ we have $\Delta(n) \geq 0$ and so the continuity of Δ assures the existence of a $p^* > 0$ with $\Delta(p^*) = 0$. ■

Proof of Theorem 2

Lemma 2 *Under assumptions A1-A8 we have*

$$\frac{\partial p^\circ}{\partial \overline{p}} = \frac{p^\circ}{\overline{p}} \cdot \varepsilon_{\ell,\overline{p}}(\tau^o, \overline{p}) \text{ and } \frac{\partial p^\circ}{\partial n} = \frac{p^\circ}{n} \text{ for } \overline{p} \in \mathrm{P}^+$$

where $\varepsilon_{\ell,\overline{p}}$ *denotes the elasticity of* ℓ *with respect to* $\overline{p}$.

Proof: This is a simple application of the envelope theorem. Because of $\overline{p} \in \mathrm{P}^+$ we have $\ell(\tau^\circ, \overline{p}) > 0$ and τ° is an interior (!) maximum of the function $f(\tau) := n \cdot \tau \cdot \ell(\tau, \overline{p}) = p^\circ(\overline{p}, n)$. By assumption A8 we may use the envelope theorem to obtain

$$\frac{\partial p^\circ}{\partial \overline{p}} = n \cdot \tau^\circ \cdot \frac{\partial \ell}{\partial \overline{p}}(\tau^\circ, \overline{p}) = \frac{p^\circ}{\overline{p}} \cdot \varepsilon_{\ell,\overline{p}}(\tau^\circ, \overline{p}) \text{ and } \frac{\partial p^\circ}{\partial n} = \tau^\circ \cdot \ell(\tau^\circ, \overline{p}) = \frac{p^\circ}{n} \qquad \blacksquare$$

Theorem 2: *Under assumptions A1-A8, if* $n_t \equiv n$ *there is a unique steady state* (p^*, τ^*). *The pension level* p^* *is strictly and inelastically increasing in* n. *In detail we have*

$$\varepsilon_{p^*,n} = \frac{1}{1 - \varepsilon_{\ell,\overline{p}}(\tau^*, p^*)}.$$

Proof: Because $p^* = p^*(n)$ is unique and $p^* \in \mathrm{P}^+$, assumption A8 yields a unique contribution rate $\tau^* = \tau^\circ(p^*, n)$. Differentiating (3) with respect to *n* yields

$$\frac{\partial p^*}{\partial n} = \frac{\partial p^\circ}{\partial \overline{p}}(p^*, n) \cdot \frac{\partial p^*}{\partial n} + \frac{\partial p^\circ}{\partial n}(p^*, n)$$

$$\overset{Lemma 2}{=} \frac{p^\circ(p^*, n)}{p^*} \cdot \varepsilon_{\ell,\overline{p}}(\tau^*, p^*) + \frac{p^\circ(p^*, n)}{n}$$

$$\overset{(3)}{=} \varepsilon_{\ell,\overline{p}}(\tau^*, p^*) \cdot \frac{\partial p^*}{\partial n} + \frac{p^*}{n}.$$

Solving for $\partial p^* / \partial n$ and dividing by p^*/n yields

$$\varepsilon_{p^*,n} = \frac{1}{1 - \varepsilon_{\ell,\overline{p}}(\tau^*, p^*)}.$$

and because $\varepsilon_{\ell,\overline{p}} < 0$ by A7(b), we have $\varepsilon_{p^*,n} \in (0,1]$. $\blacksquare$

Proof of Theorem 3

Theorem 3: *Under assumptions A1-A7 and A8', the steady-state contribution rate* τ^* *is decreasing in n.*

Proof: Because $\tau^* = \tau°(p^*)$ we have

$$\frac{\partial \tau^*}{\partial n} = \frac{\partial \tau°}{\partial \overline{p}}(p^*) \cdot \frac{\partial p^*}{\partial n} \tag{5}$$

and Theorem 2 implies that $\partial \tau^* / \partial n$ and $\partial \tau° / \partial \overline{p}(p^*)$ have the same sign. Let. $f(\tau, \overline{p}) := n \cdot \tau \cdot \ell(\tau, \overline{p})$. Differentiating the first-order condition $\partial f / \partial \tau(\tau°, \overline{p}) = 0$ of the pensioner's maximization problem with respect to $\overline{p}$ yields

$$\frac{\partial^2 f}{\partial \tau^2}(\tau°, \overline{p}) \cdot \frac{\partial \tau°}{\partial \overline{p}} + \frac{\partial^2 f}{\partial \overline{p} \partial \tau}(\tau°, \overline{p}) = 0$$

and hence we get

$$\frac{\partial \tau°}{\partial \overline{p}}(p^*) = \frac{\frac{\partial^2 f}{\partial \overline{p} \partial \tau}(\tau^*, p^*)}{-\frac{\partial^2 f}{\partial \tau^2}(\tau^*, p^*)} = \frac{\frac{\partial \ell}{\partial \overline{p}}(\tau^*, p^*) + \tau^* \cdot \frac{\partial^2 \ell}{\partial \overline{p} \partial \tau}(\tau^*, p^*)}{-\frac{\partial^2 f}{\partial \tau^2}(\tau^*, p^*)},$$

where the denominator is non-negative from the necessary second-order conditions for a maximum and the numerator is negative from A7(b) and A8'.[11]. From this and equation (5) the Theorem follows immediately. ■

[11] Because $p^* > 0$ we have $(\tau, p) \in \Lambda^+$.

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